

A photograph of a university campus featuring a row of modern, multi-story concrete buildings with balconies. In the foreground, there is a paved area with a circular water feature and a fenced-in sports field. Lush green trees and bushes are scattered throughout the scene. The sky is filled with soft, white clouds. A semi-transparent white rectangular box is overlaid on the lower-left portion of the image, containing the text 'Second class presentation'.

Second class presentation



ENERGY METHODS



In general there are four types of loading which can be applied to a material:

1. *Direct load (tension or compression)*

$$\begin{aligned}\text{Strain energy } U &= \int \frac{P^2 ds}{2AE} \quad \text{or} \quad \frac{P^2 L}{2AE} \\ &= \frac{\sigma^2 AL}{2E} = \frac{\sigma^2}{2E} \times \text{volume of bar}\end{aligned}$$

2. *Shear load*

$$\begin{aligned}\text{Strain energy } U &= \int \frac{Q^2 ds}{2AG} \quad \text{or} \quad \frac{Q^2 L}{2AG} \\ &= \frac{\tau^2}{2G} \times AL = \frac{\tau^2}{2G} \times \text{volume of bar}\end{aligned}$$

3. *Bending*

$$\text{Strain energy } U = \int \frac{M^2 ds}{2EI} \quad \text{or} \quad \frac{M^2 L}{2EI} \quad \text{if } M \text{ is constant}$$

4. *Torsion*

$$\text{Strain energy } U = \int \frac{T^2 ds}{2GJ} \quad \text{or} \quad \frac{T^2 L}{2GJ} \quad \text{if } T \text{ is constant}$$



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Deflections due to shear

Beam loading	Shear deflection	
	Rectangular-section beam	I-section beam
Cantilever—concentrated end load W'	$\frac{6WL}{5AG}$	$\frac{WL}{AG}$
Cantilever—u.d.l.	$\frac{3wL^2}{5AG}$	$\frac{wL^2}{2AG} = \frac{WL}{2AG}$
Simply supported beam—central concentrated load W	$\frac{3WL}{10AG}$	$\frac{WL}{4AG}$
Simply supported beam—concentrated load dividing span into lengths a and b	$\frac{6Wab}{5AGL}$	
Simply supported beam—u.d.l.	$\frac{3wL^2}{20AG}$	$\frac{wL^2}{8AG} = \frac{WL}{8AG}$



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11.7. Shear or distortional strain energy

11.8. Suddenly applied loads

11.9. Impact loads – axial load application

11.10. Impact loads – bending applications

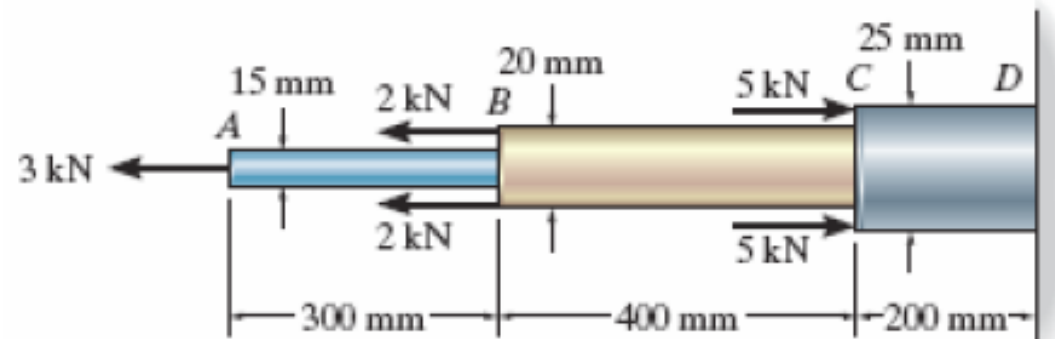
11.11. Castigliano's theorem for deflection

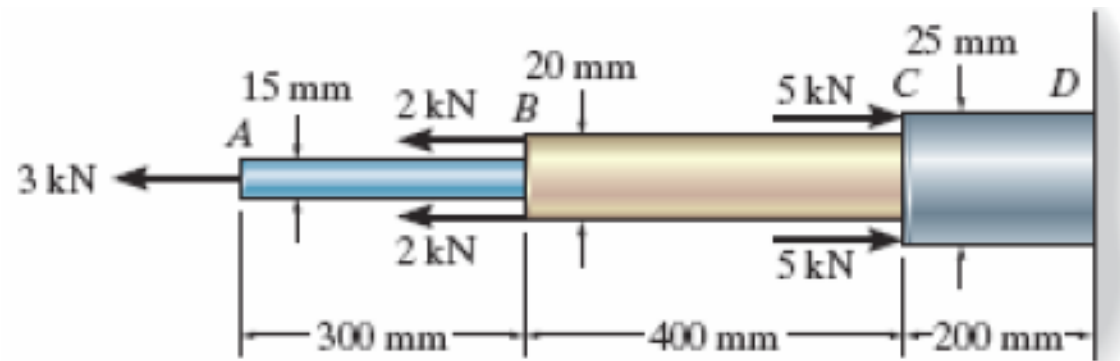


Example 1

Determine the strain energy in the rod assembly. Portion AB is steel, BC is brass, and CD is aluminum. $E_{st} = 200 \text{ GPa}$, $E_{br} = 101 \text{ GPa}$, and $E_{al} = 73.1 \text{ GPa}$.

Given: $L_{st} := 0.3\text{m}$ $d_{st} := 15\text{mm}$
 $L_{br} := 0.4\text{m}$ $d_{br} := 20\text{mm}$
 $L_{al} := 0.2\text{m}$ $d_{al} := 25\text{mm}$
 $E_{st} := 200\text{GPa}$ $P_A := 3\text{kN}$
 $E_{br} := 101\text{GPa}$ $P_B := 4\text{kN}$
 $E_{al} := 73.1\text{GPa}$ $P_C := -10\text{kN}$





Internal Axial Forces :

$$P_{st} := P_A$$

$$P_{br} := P_A + P_B$$

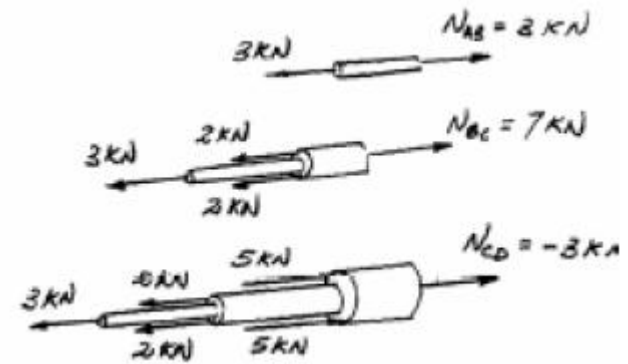
$$P_{al} := P_A + P_B + P_C$$

Axial Strain Energy: $U_i = \sum N^2 L / (2EA)$

$$U_i := \frac{P_{st}^2 \cdot L_{st}}{2 \cdot E_{st} \cdot A_{st}} + \frac{P_{br}^2 \cdot L_{br}}{2 \cdot E_{br} \cdot A_{br}} + \frac{P_{al}^2 \cdot L_{al}}{2 \cdot E_{al} \cdot A_{al}}$$

$$U_i = 0.372 \text{ J}$$

Ans



$$A_{st} := \frac{\pi \cdot d_{st}^2}{4} \quad A_{st} = 176.71 \text{ mm}^2$$

$$A_{br} := \frac{\pi \cdot d_{br}^2}{4} \quad A_{br} = 314.16 \text{ mm}^2$$

$$A_{al} := \frac{\pi \cdot d_{al}^2}{4} \quad A_{al} = 490.87 \text{ mm}^2$$



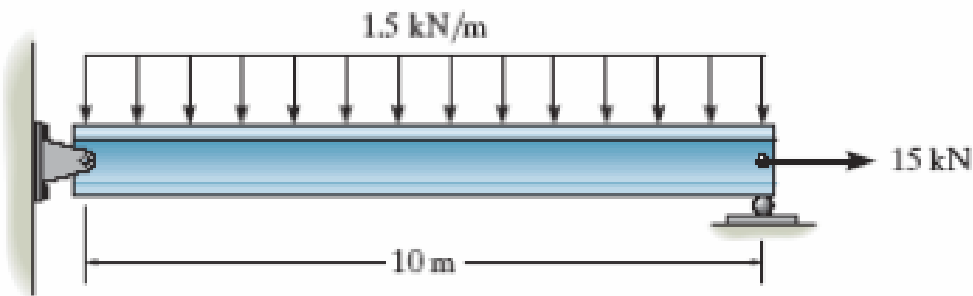
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Example 2

Determine the total axial and bending strain energy in the A-36 steel beam. $A = 2300 \text{ mm}^2$, $I = 9.5(10^6) \text{ mm}^4$.

Given: $L := 10\text{m}$ $A := 2300\text{mm}^2$
 $P := 15\text{kN}$ $I := 9.5 \cdot (10^6) \text{mm}^4$
 $E := 200\text{GPa}$ $w := 1.5 \frac{\text{kN}}{\text{m}}$

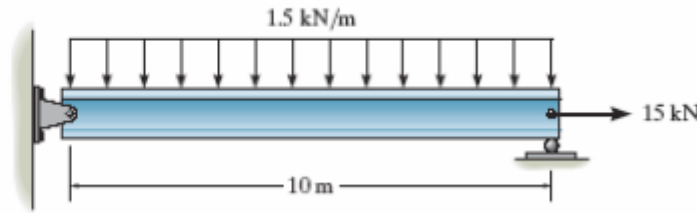
Solution:



Determine the total axial and bending strain energy in the A-36 steel beam. $A = 2300 \text{ mm}^2$, $I = 9.5(10^6) \text{ mm}^4$.

Given: $L := 10 \text{ m}$ $A := 2300 \text{ mm}^2$
 $P := 15 \text{ kN}$ $I := 9.5 \cdot (10^6) \text{ mm}^4$
 $E := 200 \text{ GPa}$ $w := 1.5 \frac{\text{kN}}{\text{m}}$

Solution:



Support Reactions :

$$+\uparrow \Sigma F_y = 0; \quad A_V + B_V - w \cdot L = 0 \quad (1)$$

$$\curvearrowleft + \Sigma M_B = 0; \quad A_V \cdot L - w \cdot L \cdot (0.5L) = 0 \quad (2)$$

Solving Eqs. (1) and (2): $A_V := 0.5w \cdot L$ $A_V = 7.5 \text{ kN}$
 $B_V := 0.5w \cdot L$ $B_V = 7.5 \text{ kN}$

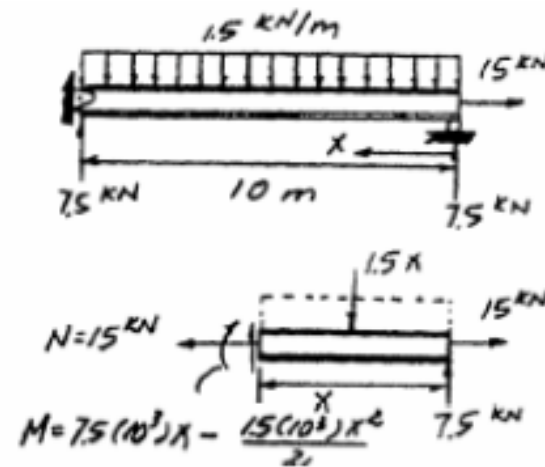
$$\rightarrow \Sigma F_x = 0; \quad A_h + P = 0$$

$$A_h := -P \quad A_h = -15 \text{ kN}$$

Internal Moment and Force Function:

$$M' = A_V \cdot x - w \cdot x \cdot (0.5x) = A_V \cdot x - 0.5w \cdot x^2$$

$$N' = -A_h$$



Total Strain Energy:

$$U_i := U_{a,i} + U_{b,i}$$

$$U_i = 495.9 \text{ J} \quad \text{Ans}$$

Axial Strain Energy: $U_i = \int_0^L N^2 / (2EA) dx$

$$U_{a,i} = \frac{1}{2E \cdot A} \int_0^L N^2 dx$$

$$U_{a,i} := \frac{1}{2E \cdot A} \int_0^L (-A_h)^2 dx$$

$$U_{a,i} = 2.4457 \text{ J}$$

Bending Strain Energy: $U_i = \int_0^L M^2 / (2EI) dx$

$$U_{b,i} = \frac{1}{2E \cdot I} \int_0^L M^2 dx$$

$$U_{b,i} := \frac{1}{2E \cdot I} \int_0^L (A_V \cdot x - 0.5w \cdot x^2)^2 dx$$

$$U_{b,i} = 493.4211 \text{ J}$$

